ECS 455 Chapter 1

Introduction

1.2 Wireless Channel (Part 1)

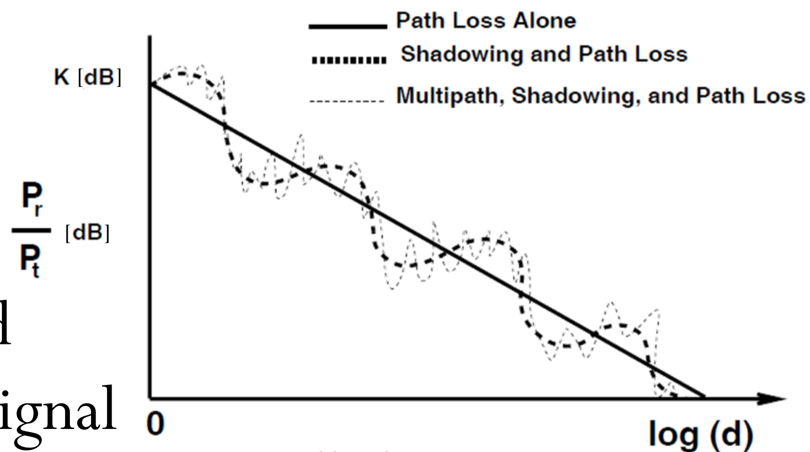
Dr.Prapun
prapun.com/ecs455

Office Hours:
BKD, 6th floor of Sirindhralai building

Tuesday	14:20-15:20
Wednesday	14:20-15:20
Friday	9:15-10:15

Wireless Channel

- **Large-scale** propagation effects
  1. Path loss
  2. Shadowing
- **Small-scale** propagation effects
  - Variation due to the constructive and destructive addition of **multipath** signal components.
  - Occur over very short distances, on the order of the signal wavelength.

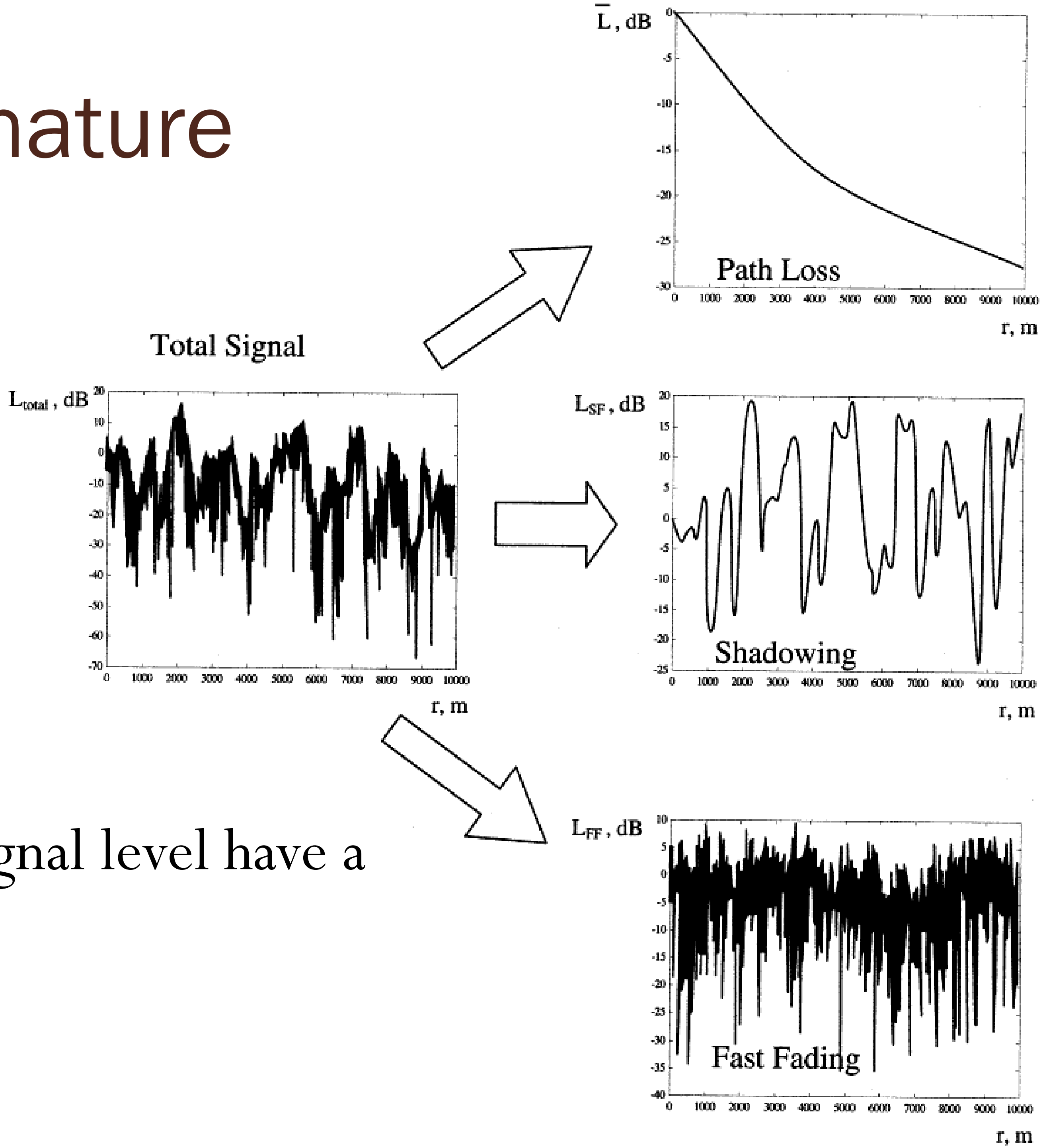


[Goldsmith, 2005, Fig 2.1]

$$\boxed{\lambda = \frac{c}{f}} \qquad c \approx 3\times10^{8}\ [\text{m/s}]$$

$f = 3$ GHz → $\lambda = 0.1$ m

Triple nature

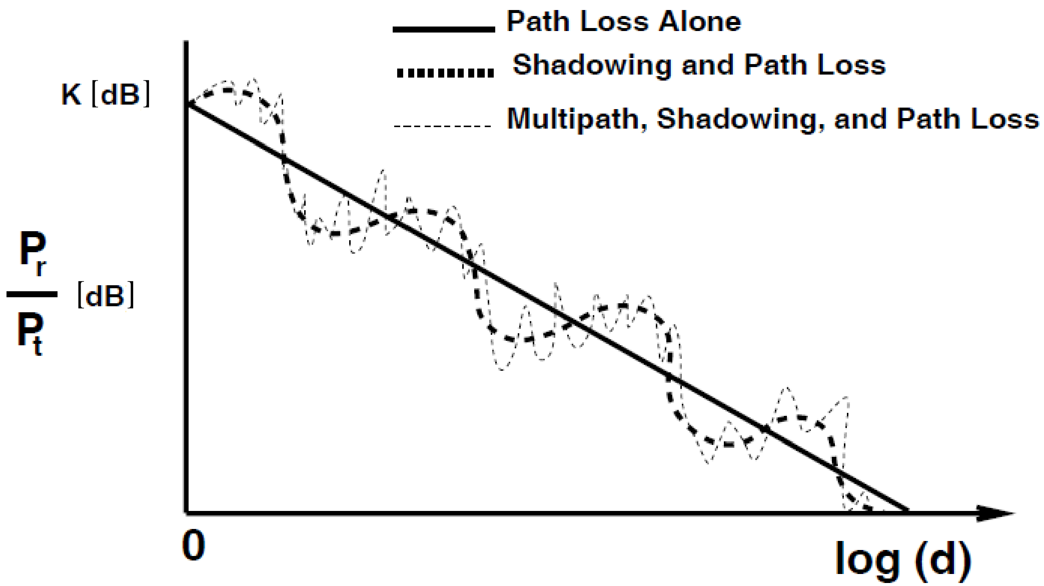


Variations of signal level have a triple nature.

[Blaunstein, 2004, Fig 12.4]

Path loss

- Caused by
  - dissipation of the power radiated by the transmitter
  - effects of the propagation channel
- Models generally assume that it is the same at a given transmit-receive distance.
- Variation occurs over **large distances** (100-1000 m)



[Goldsmith, 2005, Fig 2.1]

Path Loss (PL)

$$P_L = \frac{\text{Transmitted power}}{\text{Average received power}} = \frac{P_t}{P_r}$$

Averaged over any random variations

- **Free-Space** Path Loss Model:

$$\frac{P_r}{P_t} \propto \frac{1}{d^2}$$

  - P_r falls off inversely proportional to the square of the distance d between the Tx and Rx antennas.

- **Simplified** Path Loss Model:

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

To be discussed

(Path loss of the free-space model)

Friis Equation (Free-Space PL)

- One of the most fundamental equations in antenna theory

1 for non-directional antennas

$$\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}}\lambda}{4\pi d} \right)^2 = \left(\frac{\sqrt{G_{Tx} G_{Rx}}c}{4\pi df} \right)^2$$

- Lose more power at higher frequencies.

$$0.7\text{ GHz} \xrightarrow{} 2.4\text{ GHz} \xrightarrow{} 5\text{ GHz} \xrightarrow{} 60\text{ GHz}$$

10.7 dB loss	6.4 dB loss	21.6 dB loss
$20\log_{10}\frac{2.4}{0.7}$	$20\log_{10}\frac{5}{2.4}$	$20\log_{10}\frac{60}{5}$

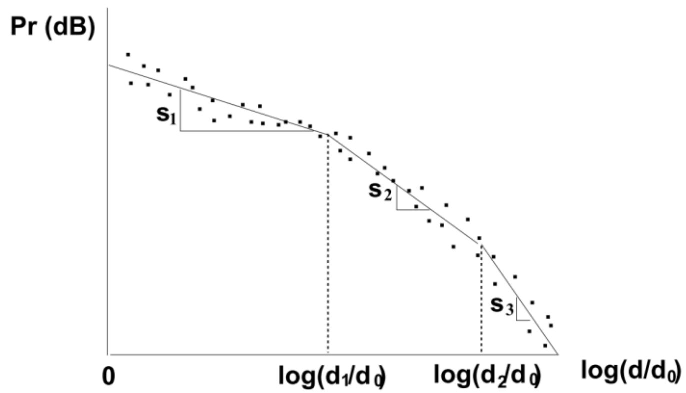
- Some of these losses can be offset by reducing the maximum operating range.
  - The remaining loss must be compensated for by increasing the antenna gain.

More Path Loss Models

- Analytical models
  - Maxwell's equations
  - Ray tracing

Prohibitive (complex, impractical)
Need to know/specify "almost everything" about the environment.

- Empirical models: Developed to predict path loss in typical environment.
  - Okumura
  - Hata
  - COST 231
    - by EURO-COST (EUROpean COoperative for Scientific and Technical research)
  - Piecewise Linear (Multi-Slope) Model
- Tradeoff: Simplified Path Loss Model



Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

$$\underset{[\text{dB}]}{10\log_{10}\frac{P_r}{P_t}} = \left(10\log_{10} K d_0^{\gamma}\right) - 10\gamma\log_{10} d$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
  - $\left(\frac{\lambda}{4\pi d_0}\right)^2$ for free-space path gain at distance d_0 assuming omnidirectional antennas
- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.

(Near-field has scattering phenomena.)

- γ is the **path loss exponent**.

[Goldsmith, 2005, Table 2.2]

Path Loss Exponent γ

- 2 in free-space model
- 4 in two-ray model
  [Goldsmith, 2005, eq. 2.17]
- Cellular: 3.5 – 4.5
  [Myung and Goodman, 2008 , p 17]
- Larger @ higher freq.
- Lower @ higher antenna heights

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Indoor Attenuation Factors

- Building penetration loss: 8-20 dB (better if behind windows)
- Attenuation between floors
  - @ 900 MHz
    - 10-20 dB when the Tx and Rx are separated by a single floor
    - 6-10 dB per floor for the next three subsequent floors
    - A few dB per floor for more than four floors
  - Typically worse at higher frequency.
- Attenuation across floors

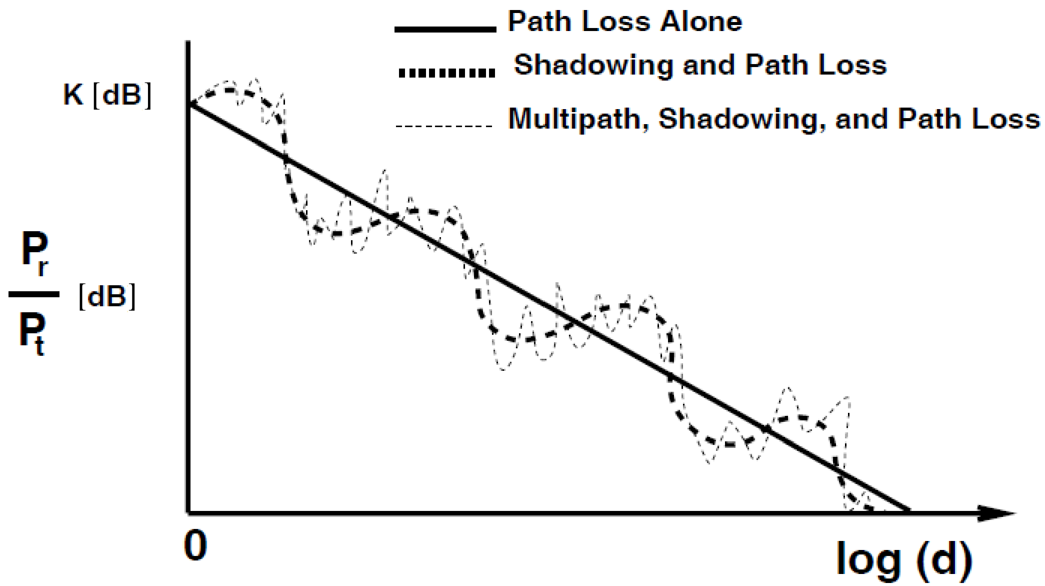
Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

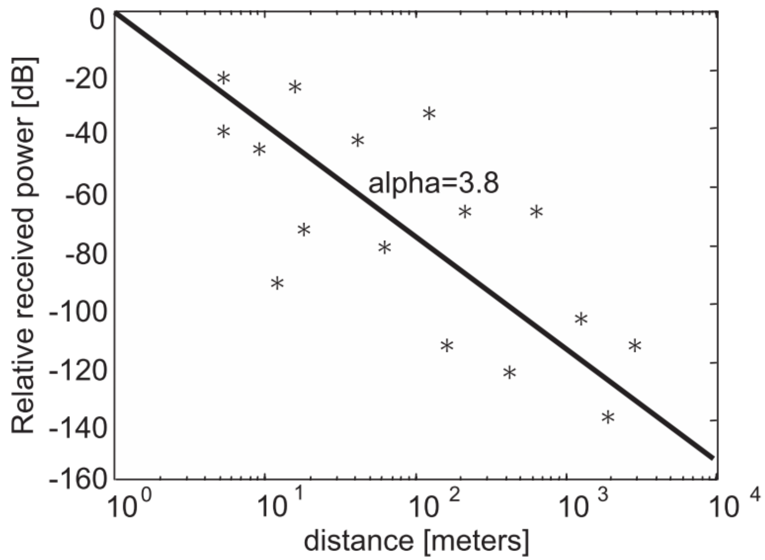
[Goldsmith, 2005, Sec. 2.5.5]

Shadowing (or Shadow Fading)

- Additional attenuation caused by **obstacles** (**large objects** such as buildings and hills) between the transmitter and receiver.
  - Think: cloud blocking sunlight
- Attenuate signal power through absorption, reflection, scattering, and diffraction.
- Variation occurs over distances proportional to the length of the obstructing object (**10-100 m** in outdoor environments and less in indoor environments).

[Goldsmith, 2005, Fig 2.1]





[Myung and Goodman, 2008, Fig 2.1]

Shadowing (Analogy)

[https://www.flickr.com/photos/pokoroto/4045274462]

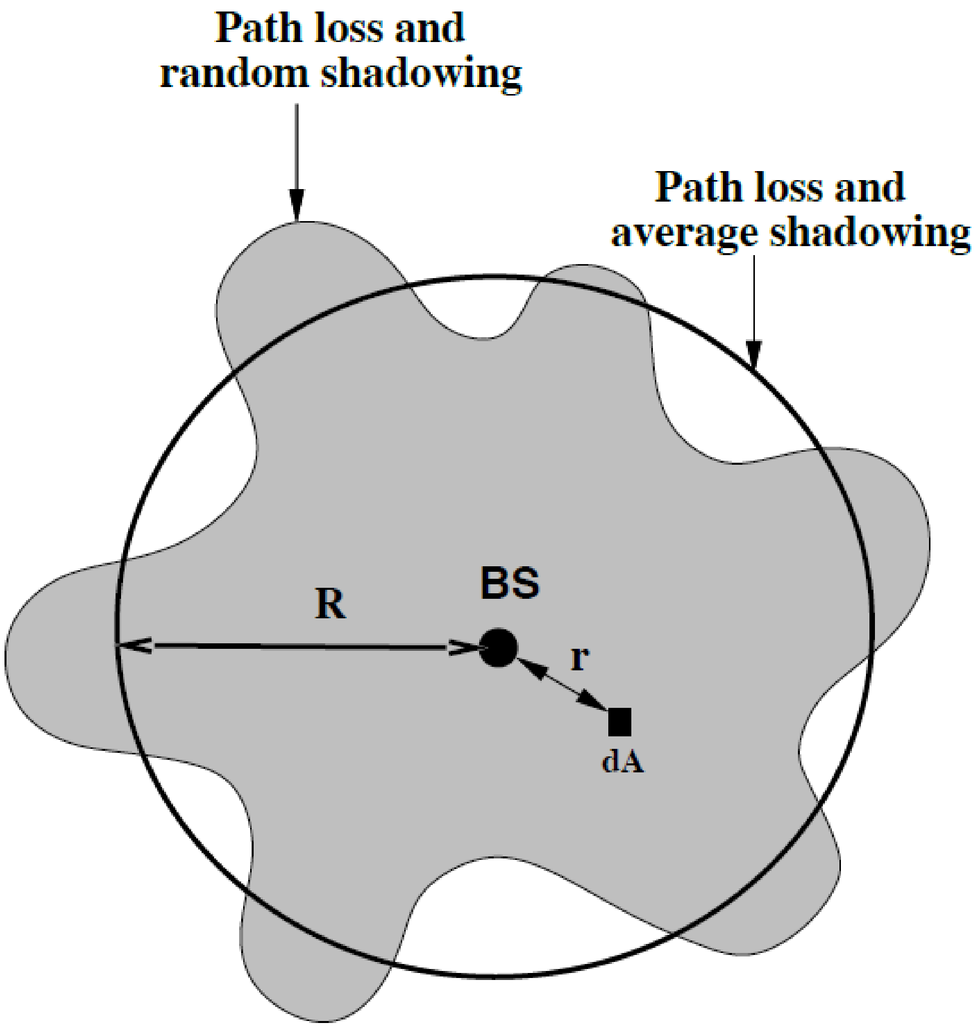
[http://spacegrant.montana.edu/MSIProject/NDVI.html]

Shadowing (Analogy)

https://brightside.me/creativity-photography/16-truly-remarkable-photos-everyone-needs-to-see-113155/#image7456160

Shadows
thousands
of miles long
cast
by clouds
on Earth's
surface.

Contours of Constant Received Power



[Goldsmith, 2005, Fig 2.10]

Log-normal shadowing

- Random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects → random variations of the received power at a given distance

4 – 13 dB with higher values in urban areas and lower ones in flat rural environments.

$$10\log_{10}\frac{P_t}{P_r} \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

in dB

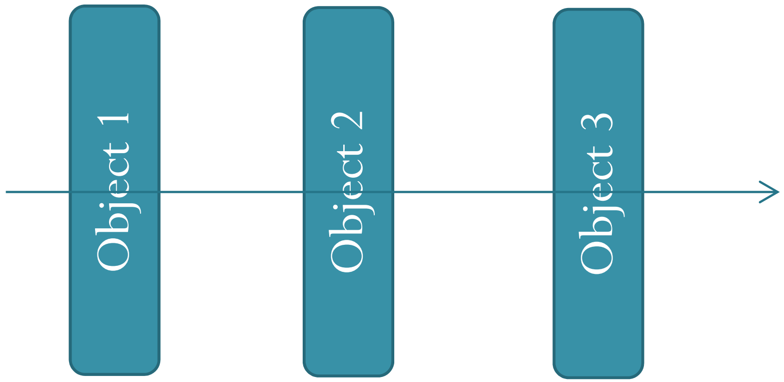
- This model has been confirmed empirically to accurately model the variation in received power in both outdoor and indoor radio propagation environments.

[Erceg et al, 1999] and [Ghassemzadeh et al, 2003]

Log-normal shadowing (motivation)

- Location, size, dielectric properties of the blocking objects as well as the changes in reflecting surfaces and scattering objects that cause the random attenuation are generally unknown

  $\Rightarrow$ statistical models must be used to characterize this attenuation.
- Assume a large number of shadowing objects between the transmitter and receiver



Without the objects, the attenuation factor is $K\left(\frac{d_0}{d}\right)^{\gamma}$.

Each object introduce extra power loss factor of α_i.

So,

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma} \prod_i \alpha_i$$

$$10\log_{10}\frac{P_r}{P_t} = 10\log_{10} K\left(\frac{d_0}{d}\right)^{\gamma} + \underbrace{\sum_i 10\log_{10}\alpha_i}_{\text{By CLT, this is approximately Gaussian}}$$

PDF of Lognormal RV

- Consider a random variable

$$R = \frac{P_t}{P_r}$$

- Suppose

$$10\log_{10} R \sim \mathcal{N}\left(\mu,\sigma^2\right)$$

Here, it should be clear why the unit of σ is in dB.

- Then,

$$f_R(r) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma}\frac{10}{\ln 10}\frac{1}{r}e^{-\frac{1}{2}\left(\frac{(10\log r)-\mu}{\sigma}\right)^2}, & r > 0 \\ 0, & \text{otherwise.} \end{cases}$$

For typical cellular environment, σ is in the range of 5-12 dB. [Proakis and Salehi, 2007, p 843]

Similar Derivation in ECS315 HW14

Problem 4. In wireless communications systems, fading is sometimes modeled by ***lognormal*** random variables. We say that a positive random variable Y is lognormal if $\ln Y$ is a normal random variable (say, with expected value m and variance σ^2).

Hint: First, recall that the ln is the natural log function (log base e). Let $X = \ln Y$. Then, because Y is lognormal, we know that $X \sim \mathcal{N}(m, \sigma^2)$. Next, write Y as a function of X.

(a) Check that Y is still a continuous random variable.

(b) Find the pdf of Y.

Solution:

Because $X = \ln(Y)$, we have $Y = e^X$. So, here, we consider $Y = g(X)$ where the function g is defined by $g(x) = e^x$.

(a) First, we count the number of solutions for $y = g(x)$. Note that for each value of $y > 0$, there is only one x value that satisfies $y = g(x)$. (That x value is $x = \ln(y)$.) For $y \le 0$, there is no x that satisfies $y = g(x)$. In both cases, the number of solutions for $y = g(x)$ is countable. Therefore, because X is a continuous random variable, we conclude that Y is also a continuous random variable.

(b) Start with $Y = e^X$. We know that exponential function gives strictly positive number. So, Y is always strictly positive. In particular, $F_Y(y) = 0$ for $y \le 0$.

Next, for $y > 0$, by definition, $F_Y(y) = P[Y \le y]$. Plugging in $Y = e^X$, we have

$$F_Y(y) = P\left[e^X \le y\right].$$

Because the exponential function is strictly increasing, the event $[e^X \le y]$ is the same as the event $[X \le \ln y]$. Therefore,

$$F_Y(y) = P[X \le \ln y] = F_X(\ln y).$$

Combining the two cases above, we have

$$F_Y(y) = \begin{cases} F_X(\ln y), & y > 0, \\ 0, & y \le 0. \end{cases}$$

Finally, we apply

$$f_Y(y) = \frac{d}{dy} F_Y(y).$$

For $y < 0$, we have $f_Y(y) = \frac{d}{dy} 0 = 0$. For $y > 0$,

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X(\ln y) = f_X(\ln y) \times \frac{d}{dy} \ln y = \frac{1}{y} f_X(\ln y). \tag{14.2}$$

Therefore,

$$f_Y(y) = \begin{cases} \frac{1}{y} f_X(\ln y), & y > 0, \\ 0, & y < 0. \end{cases}$$

At $y = 0$, because Y is a continuous random variable, we can assign any value, e.g. 0, to $f_Y(0)$. Then

$$f_Y(y) = \boxed{\begin{cases} \frac{1}{y} f_X(\ln y), & y > 0, \\ 0, & \text{otherwise.} \end{cases}}$$

Here, $X \sim \mathcal{N}(m, \sigma^2)$. Therefore,

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

and

$$\boxed{f_Y(y) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma y} e^{-\frac{1}{2}\left(\frac{\ln(y)-m}{\sigma}\right)^2}, & y > 0, \\ 0, & \text{otherwise.} \end{cases}}$$

PDF of Lognormal RV (Proof)

Suppose $c\log_b Y \sim \mathcal{N}\left(\mu,\sigma^2\right)$.

Let $X = c\log_b Y$. Note that $X = c\log_b Y = \frac{c}{\ln b}\ln\left(Y\right) = k\ln\left(Y\right)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \frac{c}{\ln b}$.

Recall, from ECS315 that to find the pdf of $Y = g(X)$ from the pdf of X, we first find the cdf of Y and then differentiate to get its pdf:

$$F_Y\left(y\right) = P\left[Y \le y\right] = P\left[e^{\frac{X}{k}} \le y\right] = P\left[X \le k\ln\left(y\right)\right] = F_X\left(k\ln\left(y\right)\right).$$

$$f_Y\left(y\right) = \frac{d}{dy}F_X\left(k\ln\left(y\right)\right) = \frac{k}{y}f_X\left(k\ln\left(y\right)\right) = \frac{1}{\sqrt{2\pi}\sigma}\frac{k}{y}e^{-\frac{1}{2}\left(\frac{k\ln\left(y\right)-\mu}{\sigma}\right)^2}.$$

PDF of Lognormal RV (Proof)

Suppose $c\log_b Y \sim \mathcal{N}\left(\mu,\sigma^2\right)$.

Let $X = c\log_b Y$. Note that $X = c\log_b Y = \dfrac{c}{\ln b}\ln\left(Y\right) = k\ln\left(Y\right)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \dfrac{c}{\ln b}$.

Alternatively, to find the pdf of $Y = g(X)$ from the pdf of X, when g is monotone, we may use the formula:

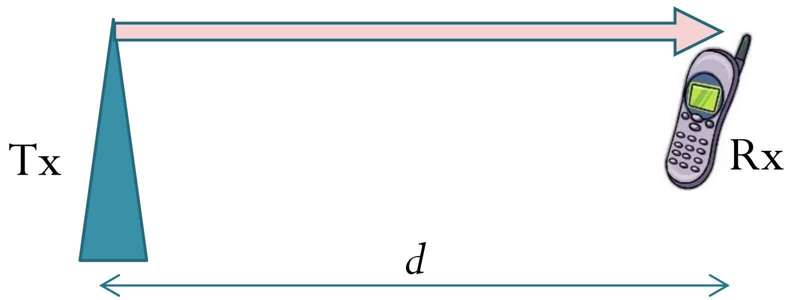
$$f_X\left(x\right)\left|dx\right| = f_Y\left(y\right)\left|dy\right| \Longrightarrow \boxed{f_Y\left(y\right) = \left|\frac{dx}{dy}\right| f_X\left(x\right)}$$

This gives $f_Y\left(y\right) = \dfrac{k}{y} f_X\left(c\log_b y\right)$ (same as what we found earlier).

Ray tracing (a prelude)

- Approximate the solution of Maxwell's equations
  - Approximate the propagation of electromagnetic waves by representing the wavefronts as simple **particles**.
  - Thus, the reflection, diffraction, and scattering effects on the wavefront are approximated using **simple geometric equations** instead of Maxwell's more complex wave equations.
- Assumption: the received waveform can be approximated by the sum of the free space wave from the transmitter plus the reflected free space waves from each of the reflecting obstacles.

$$x(t) = \sqrt{2P_t}\cos(2\pi f_c t) \qquad\qquad y(t) = ?$$



Review: Energy and Power

- Consider a signal $g(t)$.
- Total (normalized) **energy**:

Parseval's Theorem

$$\boxed{E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt} = \lim_{T\to\infty} \int_{-T}^{T} |g(t)|^2 \, dt \overset{\downarrow}{=} \int_{-\infty}^{\infty} \underbrace{|G(f)|^2}_{} df.$$

$$\boxed{\Psi_g(f) = |G(f)|^2}$$

ESD: Energy Spectral Density

- Average (normalized) **power**:

$$\boxed{P_g = \left\langle |g(t)|^2 \right\rangle} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 \, dt.$$

Review: Power Calculation

$g(t)$	$P_g = \langle \lvert g(t) \rvert^2 \rangle$
Periodic with period T_0	$\frac{1}{T_0} \int_{T_0} \lvert g(t) \rvert^2 dt$
$\sum_k a_k(t)$ where the $a_k(t)$ are **orthogonal** (e.g., do not overlap in the frequency domain)	$\sum_k P_{a_k}$

Review: Power Calculation

$g(t)$	$P_g = \langle \lvert g(t) \rvert^2 \rangle$
$\sum_k c_k e^{j2\pi f_k t}$ where the f_k are distinct	$\sum_k \lvert c_k \rvert^2$
$\sum_k a_k(t)\cos(2\pi f_k t + \phi_k)$ where the $A_k(f \pm f_k)$'s do not overlap	$\frac{1}{2}\sum_k P_{a_k}$

Power Calculation: Additional Formula

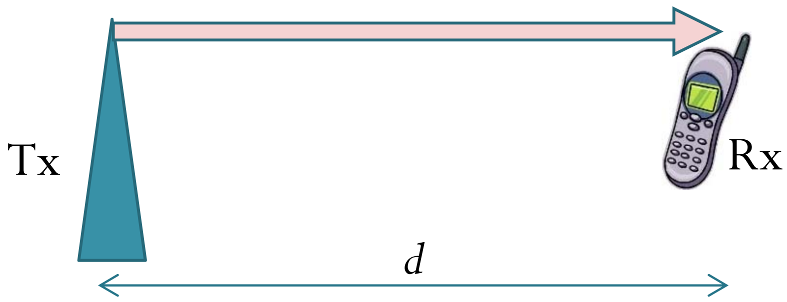
$g(t)$	$P_g = \langle \lvert g(t) \rvert^2 \rangle$
$a_1 \cos(2\pi f_c t + \phi_1) + a_2 \cos(2\pi f_c t + \phi_2)$	$= \frac{1}{2} \lvert a_1 e^{j\phi_1} + a_2 e^{j\phi_2} \rvert^2$ $= \frac{1}{2} a_1^2 + \frac{1}{2} a_2^2 + a_1 a_2 \cos(\phi_2 - \phi_1)$

Ray tracing (a revisit)

- LOS:

$$x(t)=\sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)$$



From Friis equation,

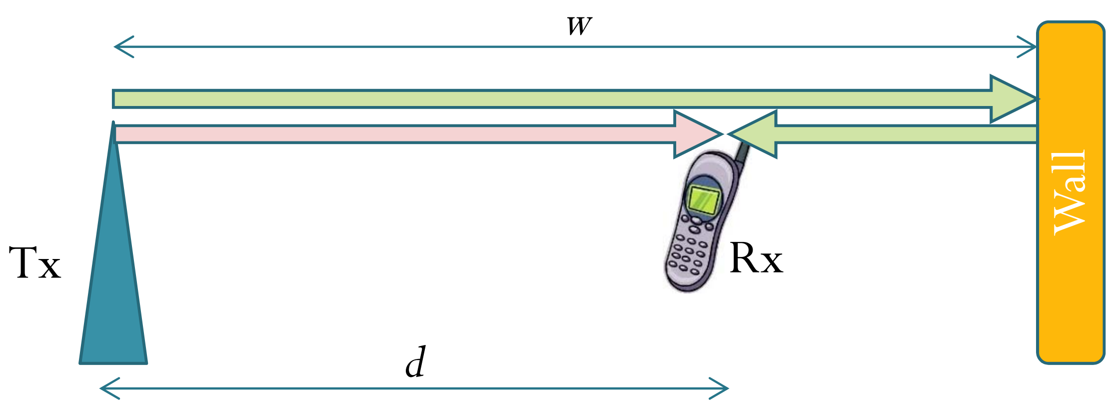
$$\alpha=\frac{\sqrt{G_{Tx}G_{Rx}}\lambda}{4\pi}.$$

- Multipath Reception

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)$$

Ex. One reflecting wall (1/4)

- There is a fixed antenna transmitting the sinusoid $x(t)$, a fixed receive antenna, and a single perfectly reflecting large fixed wall.
- Assume that the wall is very large, the reflected wave at a given point is the same (except for a sign change) as the free space wave that would exist on the opposite side of the wall if the wall were not present



$$y(t)=\sum_{k=1}^{n} R_k \frac{\alpha}{r_k}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{r_k}{c}\right)\right)$$

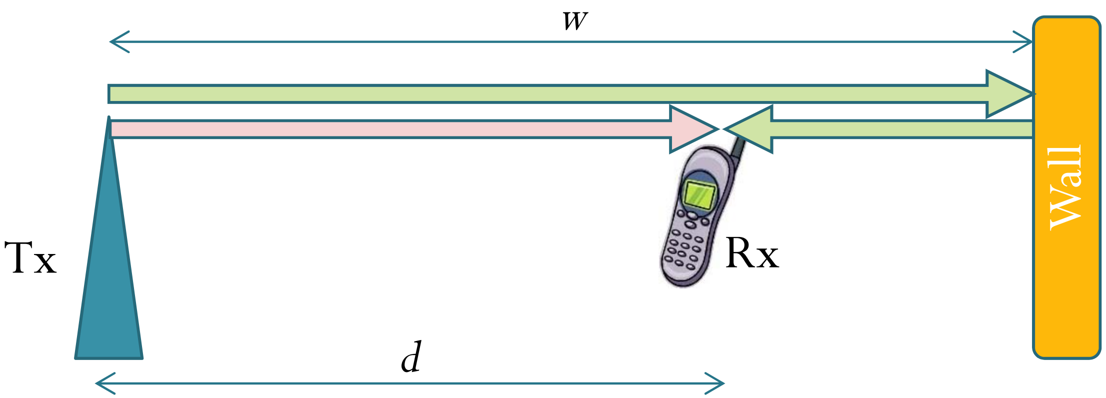
Ex. One reflecting wall (2/4)

$$x(t)=\sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)-\frac{\alpha}{\boxed{\quad}}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{\boxed{\quad}}{c}\right)\right)$$

$$=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)-\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)\right)$$

$$=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)+\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)\boxed{\quad}\right)$$



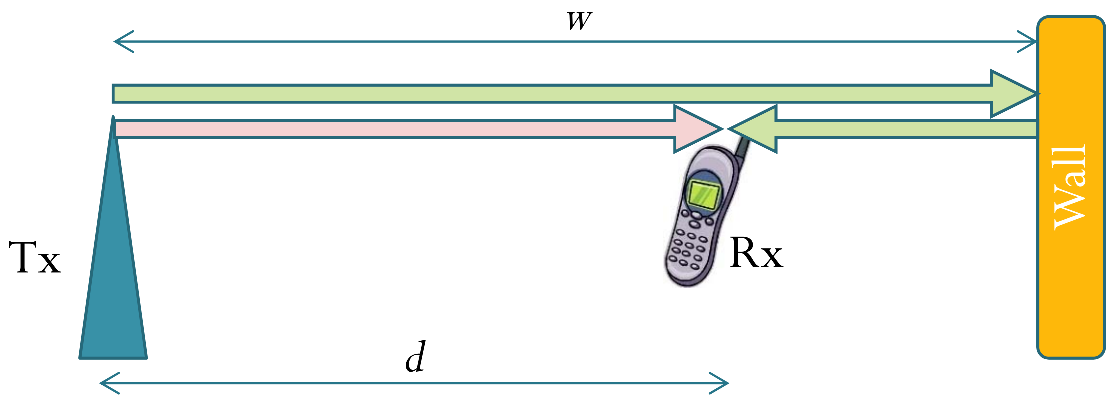
Ex. One reflecting wall (3/4)

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)+\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)-\pi\right)$$

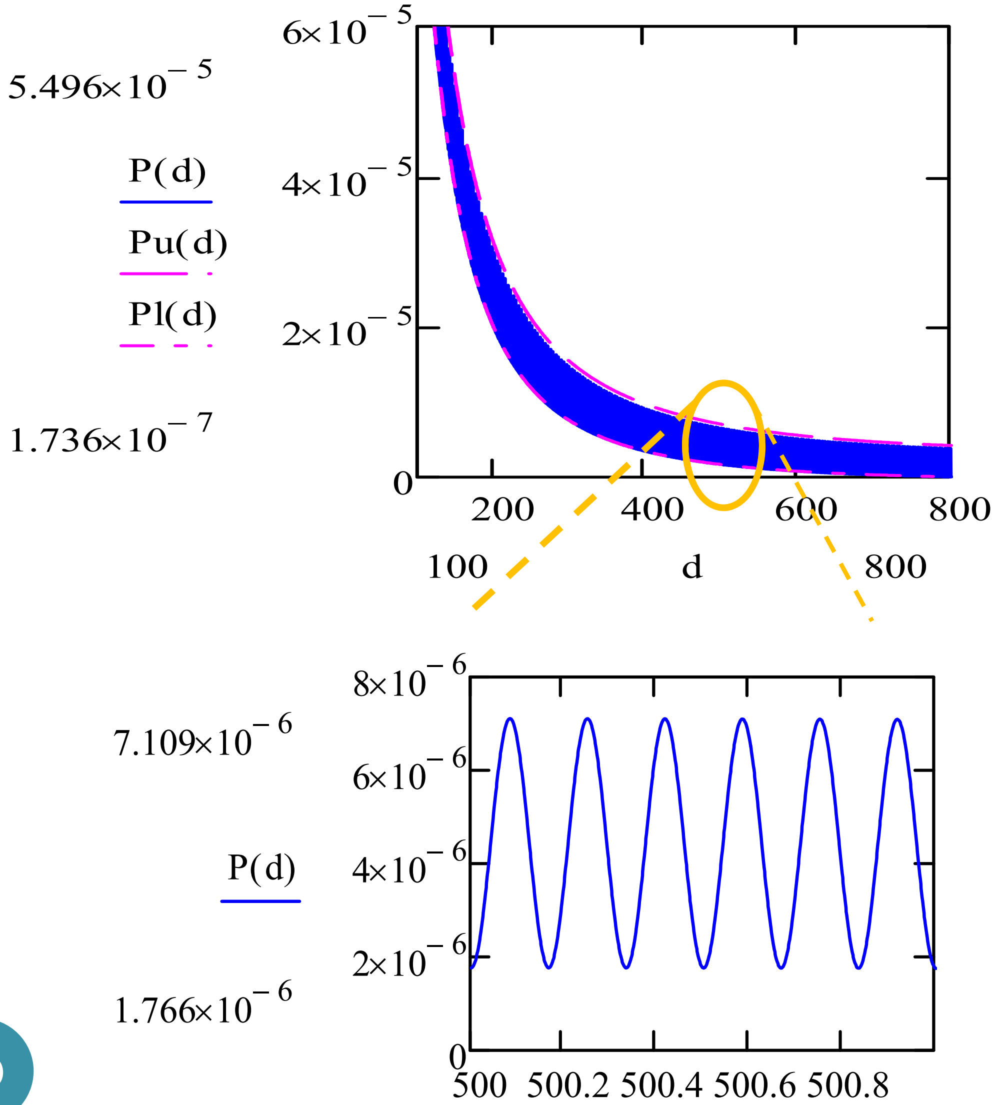
$$P_y=P_t\left(\left(\frac{\alpha}{d}\right)^2+\left(\frac{\alpha}{2w-d}\right)^2+2\frac{\alpha^2}{d(2w-d)}\cos(\Delta\phi)\right)$$

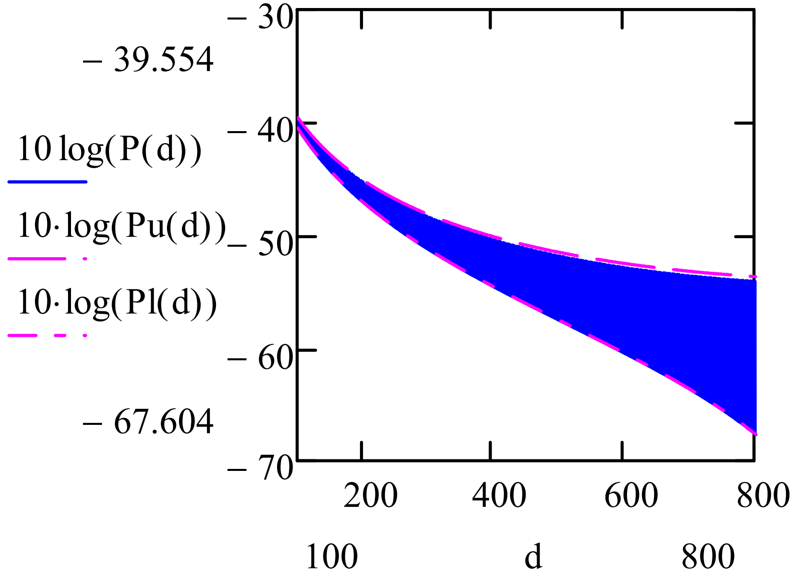
$$\Delta\phi=2\pi f_c\frac{2w-2d}{c}+\pi=2\pi\frac{1}{\lambda/2}(w-d)+\pi$$

form constructive and destructive interference pattern



Ex. One reflecting wall (4/4)





$$f = 900 \text{ MHz}$$

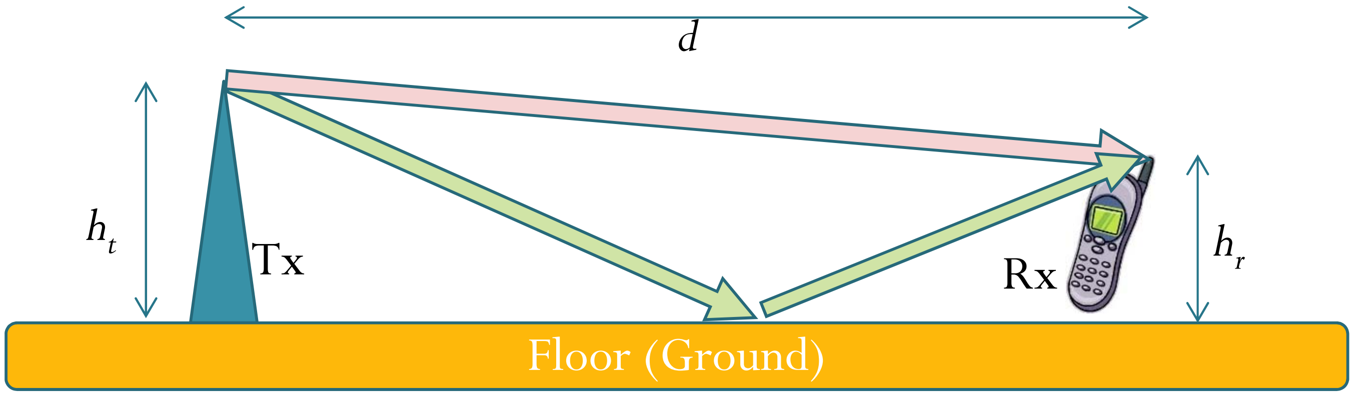
$$w = 1 \text{ km}$$

Ex. Two-Ray Model

$$\text{Delay spread} = \frac{r_2}{c} - \frac{r_1}{c}$$

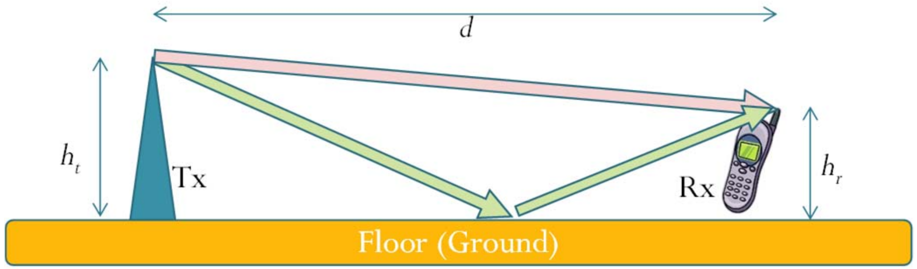
$$y(t) = \frac{\alpha}{r_1}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_2}{c}\right)\right)$$

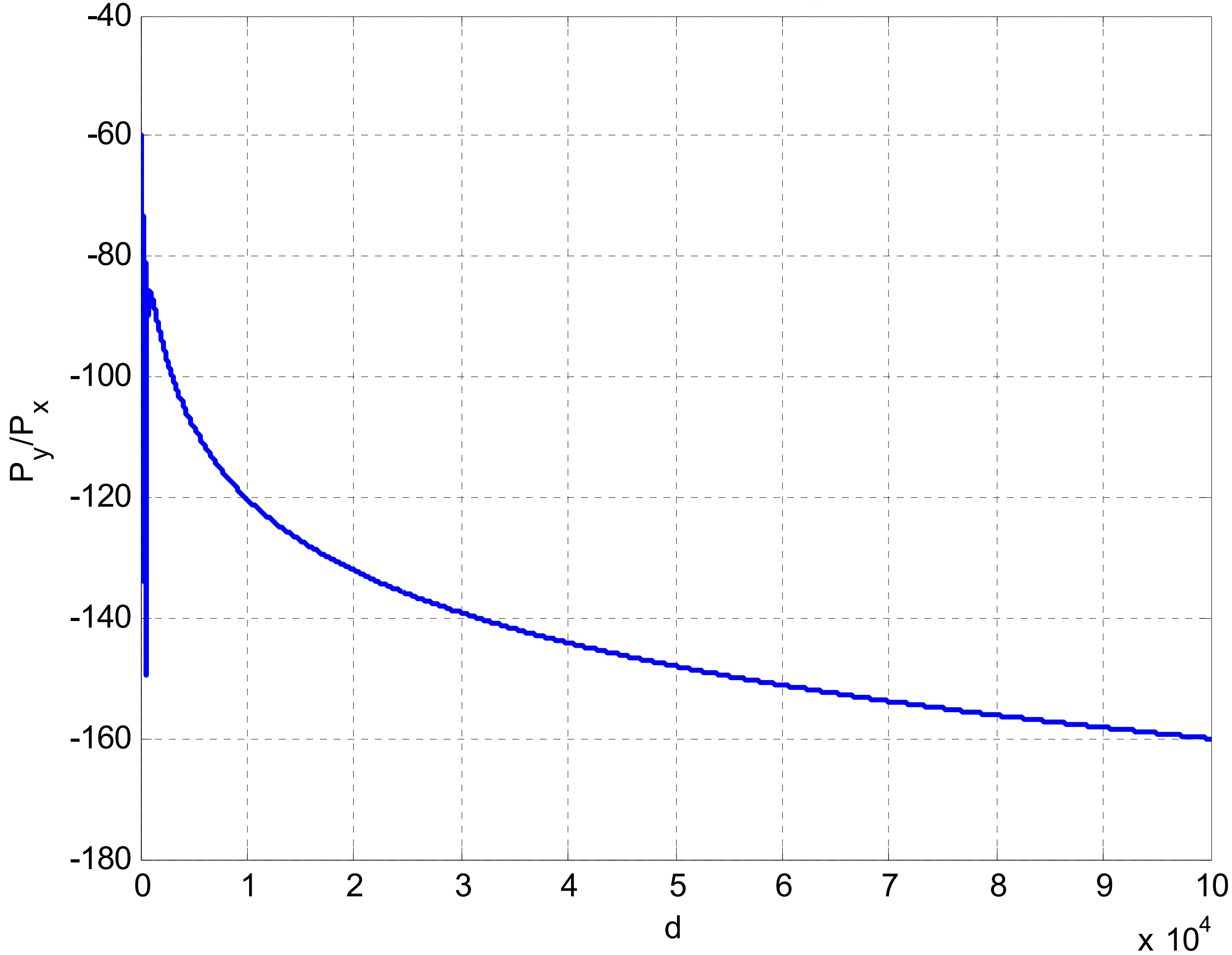
$$\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1}e^{-j2\pi f_c\frac{r_1}{c}} - \frac{\alpha}{r_2}e^{-j2\pi f_c\frac{r_2}{c}}\right|^2 = \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi f_c\frac{r_2-r_1}{c}}\right|^2$$



Assume ground reflection coefficient = -1.

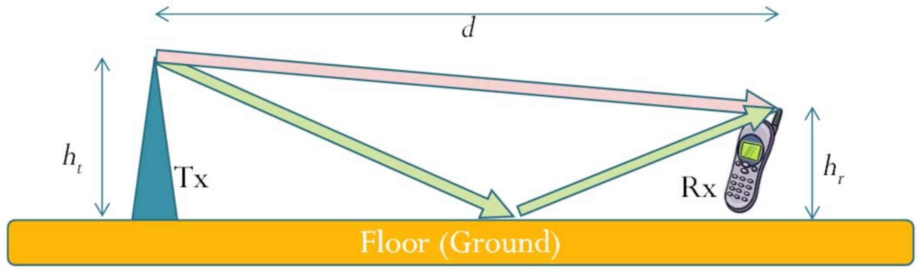
Ex. Two-Ray Model

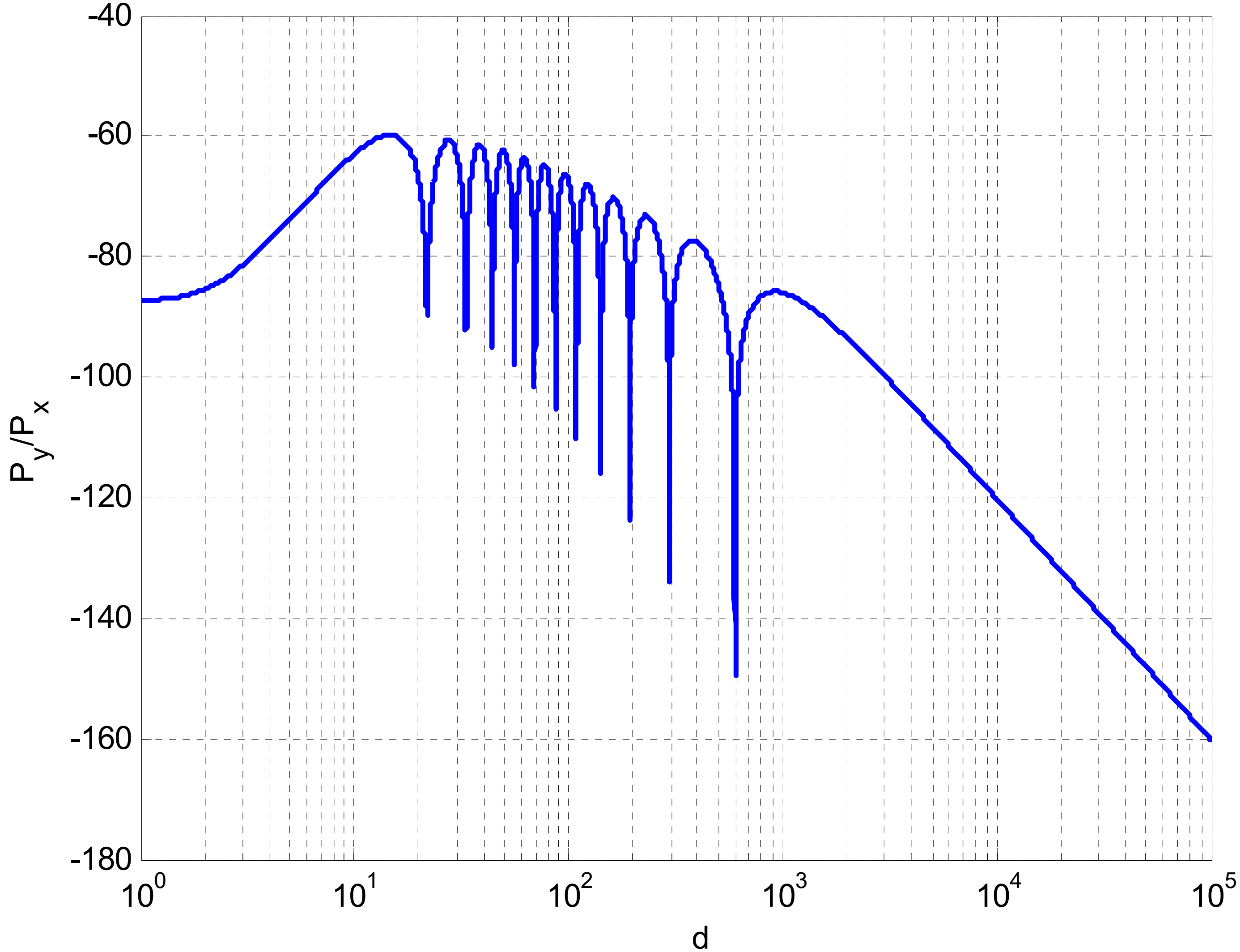




$$f = 900 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model





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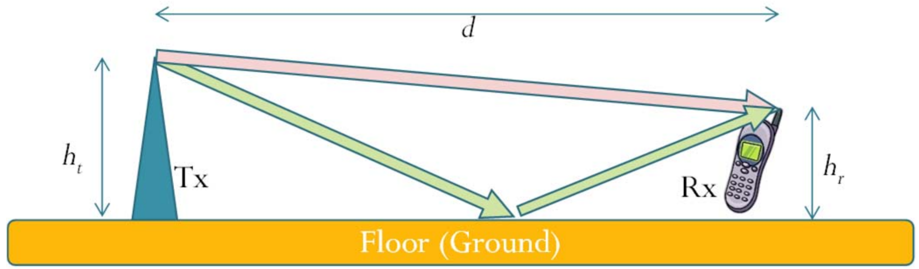
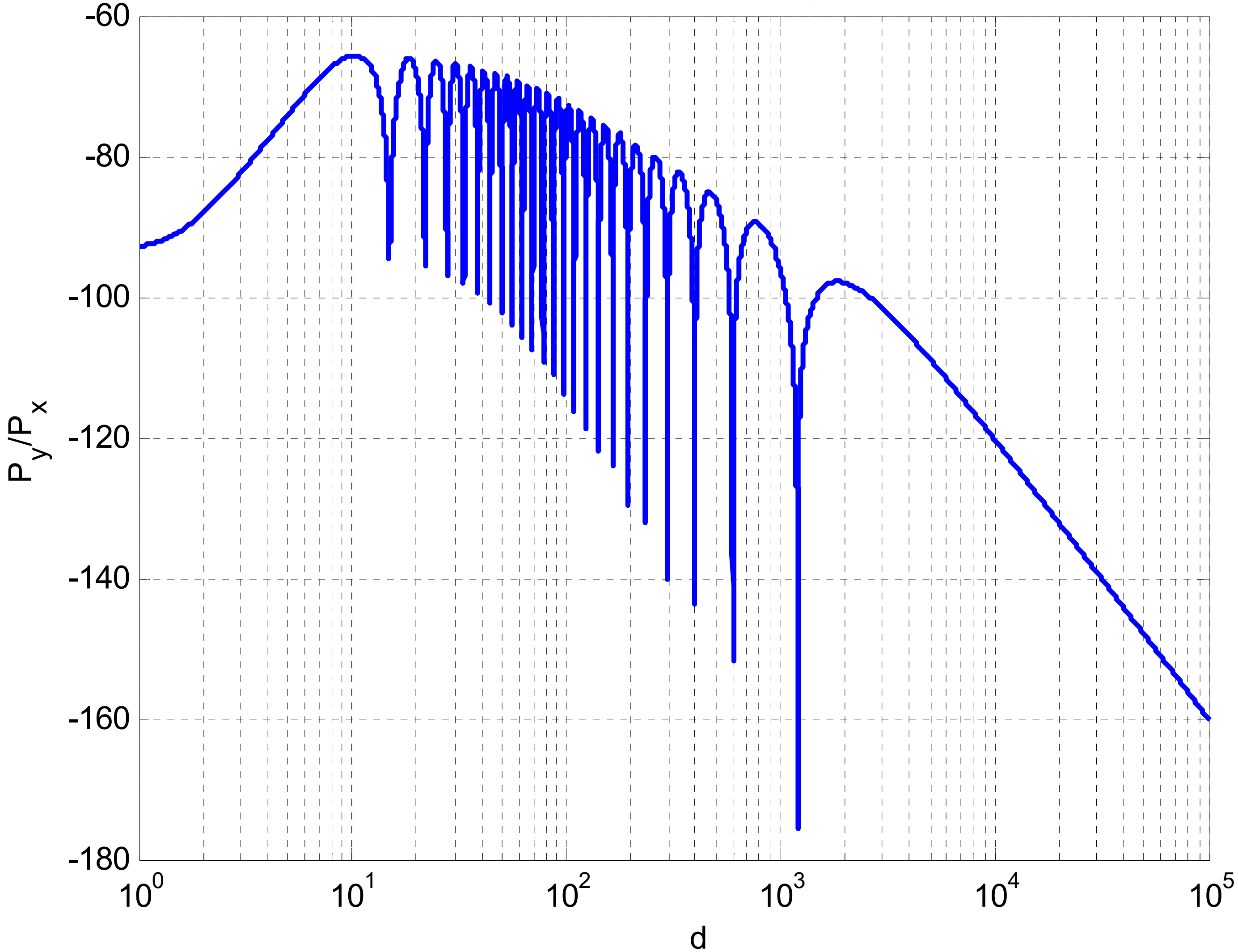
$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model





$$f = 1800 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

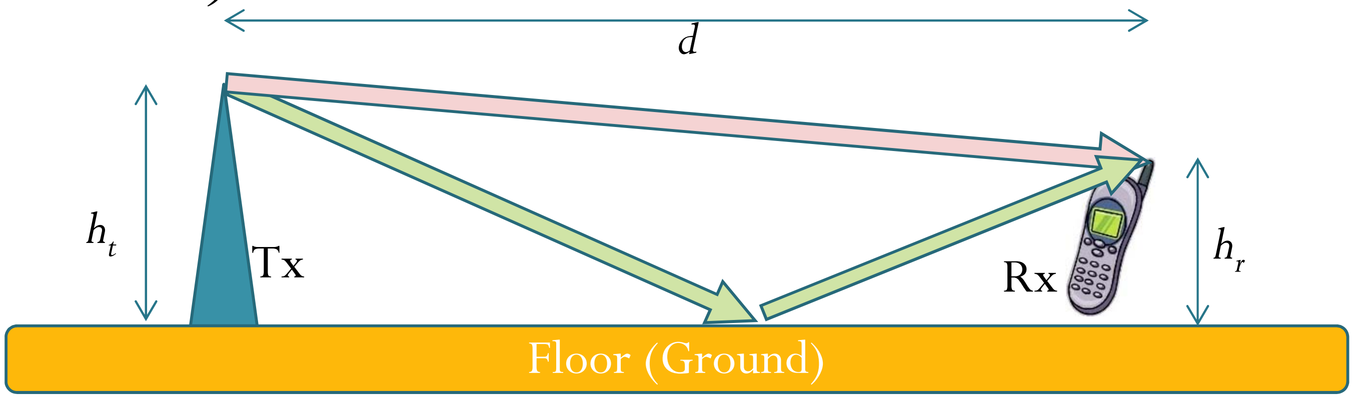
Ex. Two-Ray Model (Approximation)

$$d \gg h_t, h_r$$

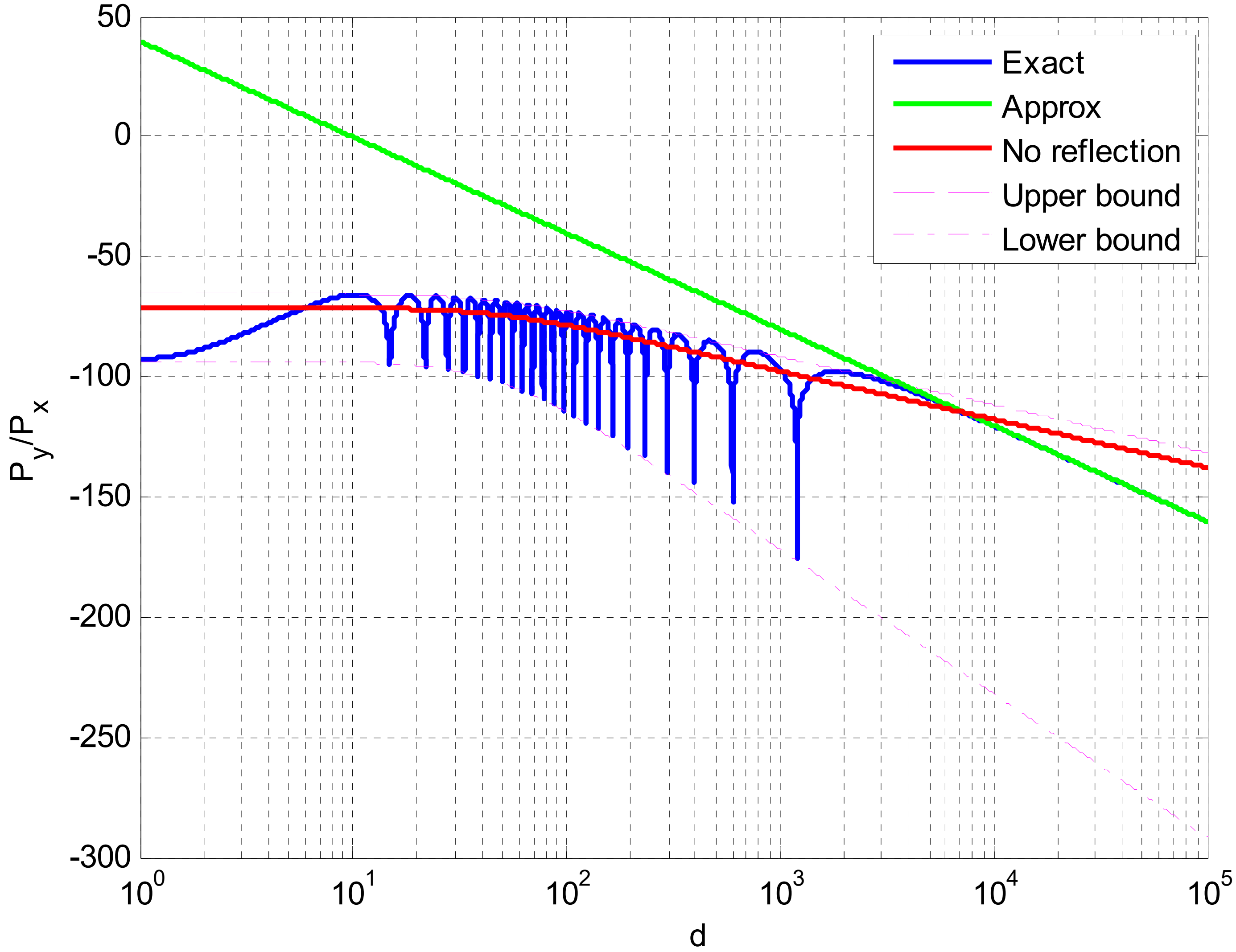
$$\frac{P_y}{P_x} \approx \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi \frac{2h_t h_r}{\lambda}} \right|^2 \approx \frac{\alpha}{d} \left| 1 - e^{-j2\pi \frac{2h_t h_r}{\lambda d}} \right|^2$$

$$\approx \left(\frac{\alpha}{d} \right)^2 \left| 1 - \left(1 - j2\pi \frac{2h_t h_r}{\lambda d} \right) \right|^2 = \frac{\alpha^2}{d^2} \left| j2\pi \frac{2h_t h_r}{\lambda d} \right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2} \right)^2$$

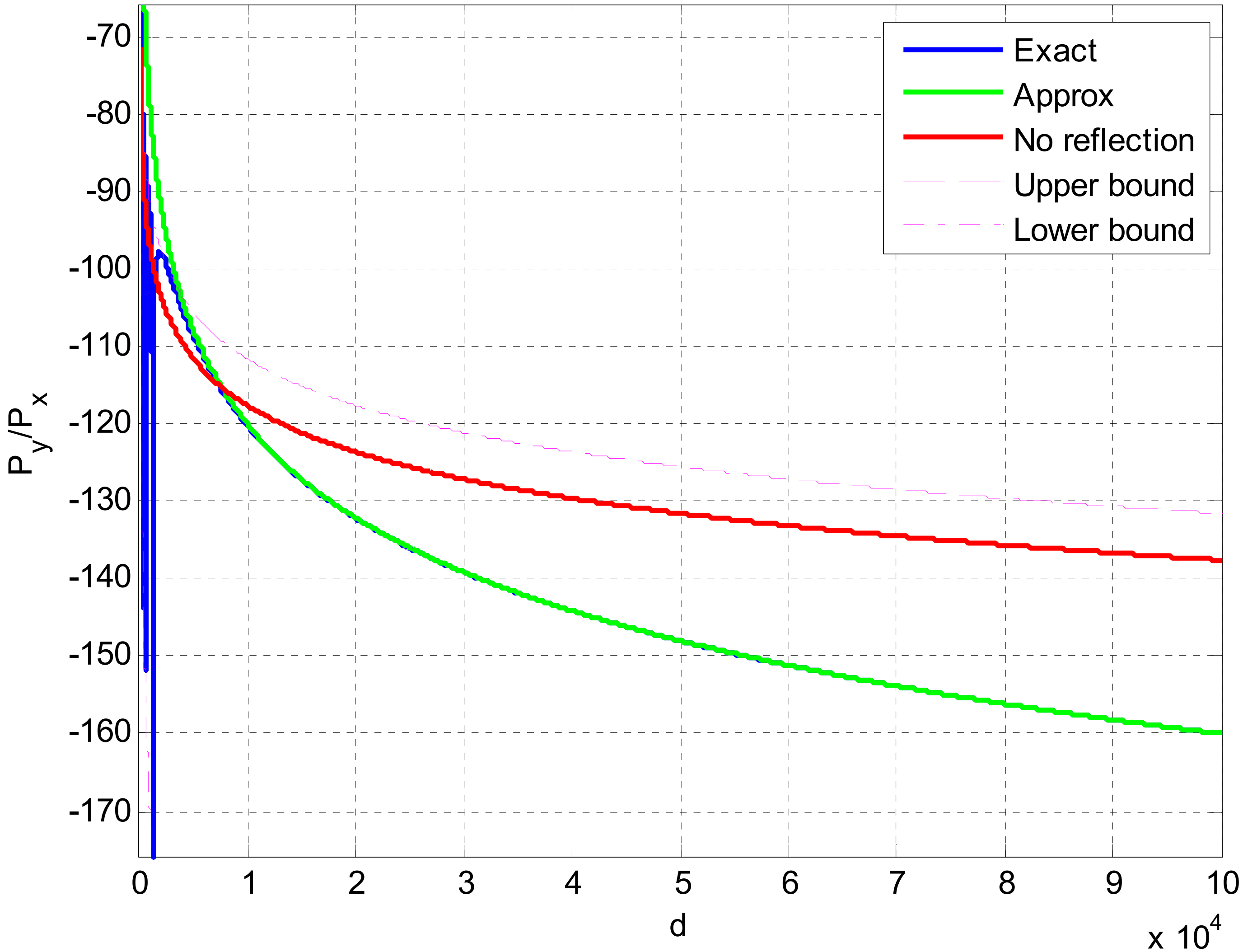
$$= \left(\frac{\sqrt{G_{Tx} G_{Rx}} h_t h_r}{d^2} \right)^2 \propto \frac{1}{d^4}$$



Ex. Two-Ray Model



Ex. Two-Ray Model



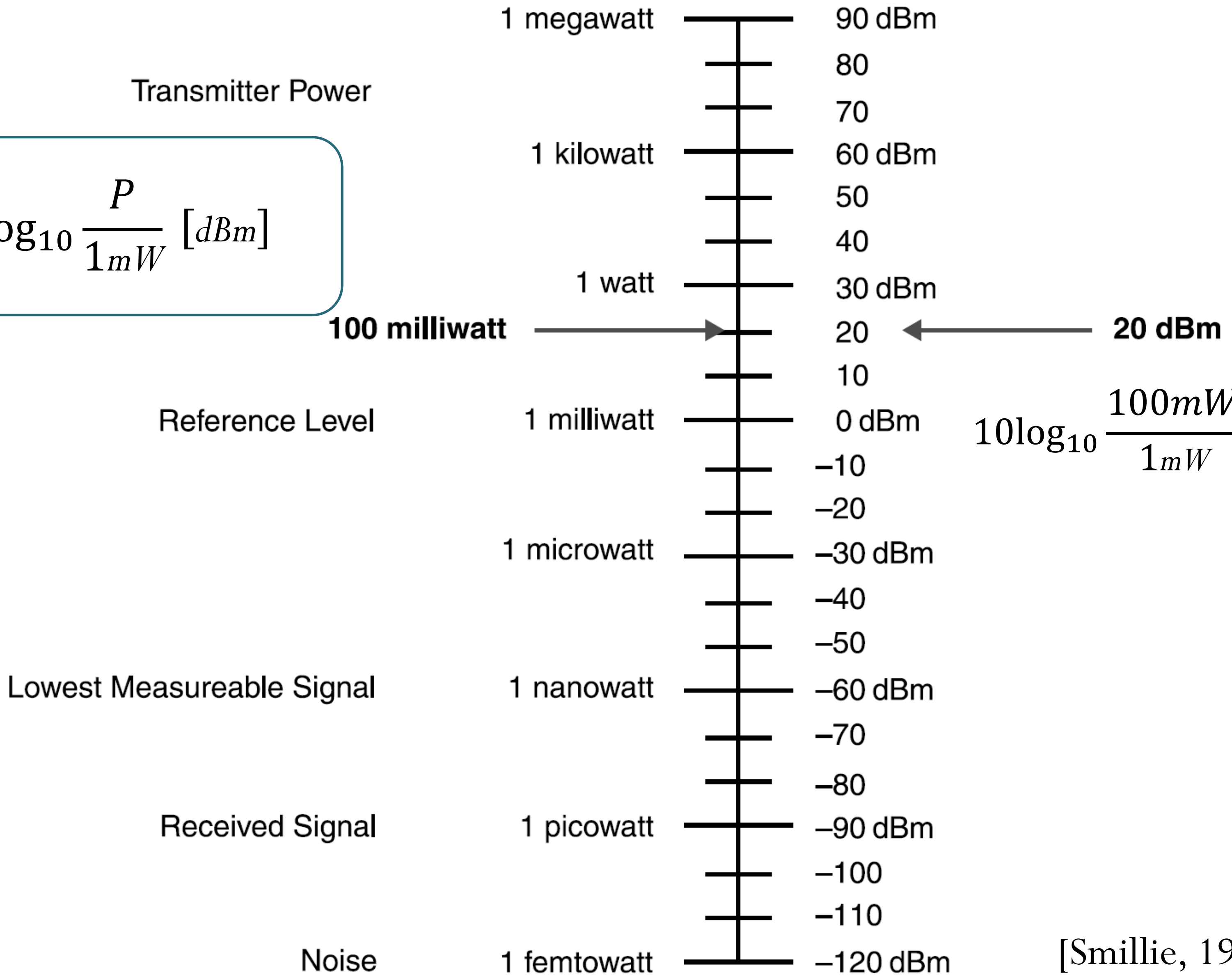
dBm

- The range of RF power that must be measured in cellular phones and wireless data transmission equipment varies from
  - hundreds of watts in base station transmitters to
  - picowatts in receivers.
- For calculations to be made, all powers must be expressed in the same power units, which is usually **milliwatts**.
  - A transmitter power of 100 W is therefore expressed as 100,000mW. A received power level of 1 pW is therefore expressed as 0.000000001mW.
- Making power calculations using decimal arithmetic is therefore complicated.
- To solve this problem, the dBm system is used.

[Scott and Frobenius, 2008, Fig 1.1]

Range of RF Power in Watts and dBm

$$P\,[W] = 10\log_{10}\frac{P}{1mW}\,[dBm]$$



$$10\log_{10}\frac{100mW}{1mW} = 20\,[dBm]$$

[Smillie, 1999, p 11]

dB and dBm

- The decibel scale expresses factors or ratios logarithmically.
- Unitless dB value
  - Represent power ratio: $10\log_{10}\frac{P_2}{P_1}$
- dB value with a unit
  - Represent the signal power itself:

$$P[\mathrm{dBW}] = 10\log_{10}\frac{P}{1\ \mathrm{W}}, \qquad P[\mathrm{dBm}] = 10\log_{10}\frac{P}{1\ \mathrm{mW}}$$

  - Note that $P[\mathrm{dBm}] = P[\mathrm{dBW}] + 30$

Remark

- Adding dB values corresponds to multiplying the underlying factors, which means multiplying the units if they are present.
- It is therefore appropriate to add unitless dB values to a dB value with a unit (such as dBm)
  - The result is still referred to that unit.
  - Ex: 17 dBm + 13 dB – 6 dB = 24 dBm
    - Correspond to 50 mW × 20 / 4 = 250 mW.

Doppler Shift: 1D Move

- At the transmitter, suppose we have

$$\sqrt{2P_t}\cos\left(2\pi f_c t+\phi\right)$$

- At distance r (far enough), we have

Time to travel a distance of r

$$\frac{\alpha}{r}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{r}{c}\right)+\phi\right)$$

- If moving, r becomes $r(t)$.
- If moving ***away*** at a constant velocity v, then $r(t)=r_0+vt.$

$$\frac{\alpha}{r(t)}\cos\left(2\pi f_c\left(t-\frac{r_0+vt}{c}\right)+\phi\right)=\frac{\alpha}{r(t)}\cos\left(2\pi\left(f_c-f_c\frac{v}{c}\right)t-2\pi f_c\frac{r_0}{c}+\phi\right)$$

Frequency shift

$$\Delta f=\frac{v}{\lambda}$$

Review: Instantaneous Frequency

For a generalized sinusoid signal

$$A\cos\left(\theta\left(t\right)\right),$$

the **instantaneous frequency** at time t is given by

$$f\left(t\right)=\frac{1}{2\pi}\frac{d}{dt}\theta\left(t\right).$$

When $\theta\left(t\right)=2\pi f_c\left(t-\frac{r\left(t\right)}{c}\right)+\phi,$

$$f\left(t\right)=\frac{1}{2\pi}\frac{d}{dt}\theta\left(t\right)=f_c-\frac{f_c}{c}\frac{d}{dt}r\left(t\right)=f_c-\frac{1}{\lambda}\frac{d}{dt}r\left(t\right)$$

Frequency shift

Big Picture

Transmission impairments in cellular systems

Physics of radio propagation	Attenuation (Path Loss) Shadowing Doppler shift Inter-symbol interference (ISI) Flat fading Frequency-selective fading
Extraneous signals	Co-channel interference Adjacent channel interference Impulse noise White noise
Transmitting and receiving equipment	White noise Nonlinear distortion Frequency and phase offset Timing errors

[Myung and Goodman, 2008, Table 2.1]